Review Midterm 1

Wednesday, April 26, 2023 8:48 AM

review sequences: $(a_n) = a_1, a_2, a_3 \ldots$

- write terms
- decide
  - convergent : $\lim_{n\to\infty} a_n$ (hierarchy growth, MCT, comparison test)
  - divergent
    - oscillating
    - $a_n \to \infty$ or $-\infty$

    (hierarchy growth, comparison test)

hierarchy growth: $\ln(n) \ll n^k \ll a^n \ll n! \ll n^n$

MCT: increasing & bounded above / decreasing & bounded below

comparison test: $b_n > a_n$ / $\lim_{n\to\infty} b_n$ = converges $\to$ $\lim_{n\to\infty} a_n$ = converges

ex) $a_n = 1/2^n \quad n \geq 0$

1) write $a_1, a_2, a_3, a_4 \ldots$

$a_0 = 1/2^0 = 1$

$a_1 = 1/2^1 = 1/2$

$a_2 = 1/2^2 = 1/4$

$a_3 = 1/2^3 = 1/8$

2) show (a_n) decreasing $(a_{n+1} < a_n)$

$$\frac{1}{2^{n+1}} < \frac{1}{2^n} \longrightarrow 1 < \frac{2^{n+1}}{2} \longrightarrow 1 < 2 \checkmark$$

3) show $a_n = 1/2^n$ converges

1. compute directly
2. part 2 shows decreasing

   also $a_n = \frac{1}{2^n} \geq 0$, bounded below

   $\Big\}$ MCT = converges

* recursive $\to$ possibly use L as value bounded by *

review series : $\sum_{n=1}^{\infty} a_n$

- decide convergent vs divergent
  - integral test
  - ratio test $\to$ with $n!$
  - root test $\to$ with $\blacksquare^n$
  - alternating series test $\to$ $\sum (-1)^n \cdot a_n$
    - $a_n \to 0$
    - $a_{n+1} < a_n$
    - $a_n \geq 0$
  - comparison test $\to$ $\sum \frac{1}{n^2+19}$

two special types:

- p-series : $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$ $\to$ convergent if $p > 1$
- geometric series : $\sum_{n=1}^{\infty} r^n = \frac{a}{1-r}$ $\to$ if $|r| < 1$ $\to$ convergent

  (absolute convergence)

review taylor series :

- taylor expansion : $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

  where $a_n = \frac{f^{(n)}(a)}{n!}$

ex) given $f(x) = \sin(x)$ & $a = 0$

$f(0) = \sin(0) = 0$

$f'(0) = \cos(0) = 1$

• taylor expansion : $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

where $a_n = \frac{f^{(n)}(a)}{n!}$

• compute $f(x)$, $f'(x)$, $f''(x)$ ... (a typically $= 0$)

$f(0) = \sin(0) = 0$

$f'(0) = \cos(0) = 1$

$f''(0) = -\sin(0) = 0$

$f'''(0) = -\cos(0) = -1$

$\vdots$

* truncation of degree 5 $\longrightarrow x^5$ *

* taylor can multiply & plug in variables *

* taylor of polynomial = polynomial *

* memorize : $\sin(x)$, $\cos(x)$, e^x, $\ln(x)$ *